Class-XII Matrix. Ex. 3.2 10-07-2020

Q.17. If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then find K so that $A^2 = KA - 2I$

Soln: It is given that $A^2 = KA - 2I \Rightarrow AA = KA - 2I$

$$\Rightarrow \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = K\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix} = \begin{bmatrix} 3K & -2K \\ 4K & -2K \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3K-2 & -2K-0 \\ 4K-0 & -2K-2 \end{bmatrix}$$

By definition of equality of matrix, we get

$$\Rightarrow \begin{matrix} 3K-2=1, & -2K-0=-2 \\ 4K-0=4, & -2K-2=-4 \end{matrix}$$

Now, $3K = 3$ $\qquad -2K = -2$

$\Rightarrow K = \frac{3}{3} = 1$ $\qquad \Rightarrow K = \frac{-2}{-2} = 1$

And $4K - 0 = 4$ $\qquad -2K-2=-4$

$\Rightarrow 4K = 4$ $\qquad \Rightarrow -2K = -4+2$

$\Rightarrow K = \frac{4}{4}$ $\qquad \Rightarrow -2K = -2$

$\Rightarrow K = 1$ $\qquad \Rightarrow K = \frac{-2}{-2}$

$\qquad \Rightarrow K = 1$

Q.18 If $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2, show that $I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

Soln: It is given that $A = \begin{bmatrix} 0 & -t \\ t & 0 \end{bmatrix}$, where $t = \tan\left(\frac{\alpha}{2}\right)$

Now, $\cos\alpha = \frac{1-\tan^2\left(\frac{\alpha}{2}\right)}{1+\tan^2\left(\frac{\alpha}{2}\right)} = \frac{1-t^2}{1+t^2}$ and $\sin\alpha = \frac{2\tan\left(\frac{\alpha}{2}\right)}{1+\tan^2\left(\frac{\alpha}{2}\right)} = \frac{2t}{1+t^2}$

~~RHS = $(I-A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} = \left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & -t \\ t & 0 \end{pmatrix}\right]$~~

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$$R.H.S = (I-A)\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix} = \left[\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} - \begin{pmatrix}0 & -t\\ t & 0\end{pmatrix}\right]\begin{bmatrix}\frac{1-t^2}{1+t^2} & \frac{-2t}{1+t^2}\\ \frac{2t}{1+t^2} & \frac{1-t^2}{1+t^2}\end{bmatrix}$$

$$= \begin{bmatrix}1 & t\\ -t & 1\end{bmatrix}\begin{bmatrix}\frac{1-t^2}{1+t^2} & \frac{-2t}{1+t^2}\\ \frac{2t}{1+t^2} & \frac{1-t^2}{1+t^2}\end{bmatrix} = \begin{bmatrix}\frac{1-t^2+2t^2}{1+t^2} & \frac{-2t+t(1-t^2)}{1+t^2}\\ \frac{-t(1-t^2)+2t}{1+t^2} & \frac{2t^2+1-t^2}{1+t^2}\end{bmatrix}$$

$$= \begin{bmatrix}\frac{1+t^2}{1+t^2} & \frac{-2t+t-t^3}{1+t^2}\\ \frac{-t+t^3+2t}{1+t^2} & \frac{2t^2+1-t^2}{1+t^2}\end{bmatrix} = \begin{bmatrix}\frac{\cancel{1+t^2}}{\cancel{1+t^2}} & \frac{-t\cancel{(1+t^2)}}{\cancel{1+t^2}}\\ \frac{t\cancel{(1+t^2)}}{\cancel{1+t^2}} & \frac{\cancel{1+t^2}}{\cancel{1+t^2}}\end{bmatrix} = \begin{bmatrix}1 & -t\\ t & 1\end{bmatrix}$$

$$L.H.S = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} + \begin{bmatrix}0 & -t\\ t & 0\end{bmatrix} = \begin{bmatrix}0+1 & -t+0\\ t+0 & 0+1\end{bmatrix} = \begin{bmatrix}1 & -t\\ t & 1\end{bmatrix} = R.H.S$$

putting the value of t on both sides, we get

$$\begin{bmatrix}1 & -\tan\left(\frac{\alpha}{2}\right)\\ \tan\left(\frac{\alpha}{2}\right) & 1\end{bmatrix} = \begin{bmatrix}1 & -\tan\left(\frac{\alpha}{2}\right)\\ \tan\left(\frac{\alpha}{2}\right) & 1\end{bmatrix}$$

Q.19. A trust fund has ₹30000 that must be invested in two different types of bonds. The first bond pays 5% interest per year and the second ~~bond~~ bond pays 7% interest per year. Using matrix multiplication, determine how to divide ₹30000 among the two types of bonds, if the trust fund must obtain an annual total interest of (a) ₹1800 (b) ₹2000

Soln: Let the amount invested in first type of bonds is ₹x, then that invested in second type of bonds will be ₹$(30000-x)$

(a) According to the given condition

$$\begin{bmatrix}x & 30000-x\end{bmatrix}\begin{bmatrix}\frac{5}{100}\\ \frac{7}{100}\end{bmatrix} = \begin{bmatrix}1800\end{bmatrix}$$

$$\Rightarrow \left[\frac{5x}{100} + \frac{(30000-x)7}{100}\right] = [1800]$$

$$\Rightarrow \frac{5x + 210000 - 7x}{100} = 1800$$

$$\Rightarrow 210000 - 2x = 180000 \Rightarrow 30000 = 2x \Rightarrow x = \frac{30000}{2} = 15000$$

Hence, the amounts invested in the two types of bonds are respectively ₹15000 and ₹(30000−15000) = ₹15000

(b) According to the given condition,

$$\begin{bmatrix} x & 30000-x \end{bmatrix} \begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = [2000]$$

$$\Rightarrow \left[\frac{5x}{100} + \frac{(30000-x)7}{100}\right] = [2000]$$

Hence, the amounts invested in two types of bonds are respectively ₹5000 and ₹(30000−5000) = ₹25000.

Q.20 The bookshop of a particular school has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are ₹80, ₹60 and ₹40 each respectively. Find the total amount the bookshop will receive from selling the books using matrix algebra.

Soln: Let $A = \begin{bmatrix} 10\times12 & 8\times12 & 10\times12 \end{bmatrix}$

and $B = \begin{bmatrix} 80 & 60 & 40 \end{bmatrix}^T = \begin{bmatrix} 80 \\ 60 \\ 40 \end{bmatrix}$

$$\therefore AB = \begin{bmatrix} 120 & 96 & 120 \end{bmatrix} \begin{bmatrix} 80 \\ 60 \\ 40 \end{bmatrix} = \begin{bmatrix} 120\times80 + 96\times60 + 120\times40 \end{bmatrix}$$

$$= [9600 + 5760 + 4800]_{1\times1} = [20160]_{1\times1}$$

∴ Amount received by the bookseller = ₹20160

Matrix Ex 3.2. 10-07-2020

Q.21. The restrictions on n, k and p so that $PY + WY$ will be

(a) $k=3$, $p=n$ (b) k is arbitrary, $p=2$

(c) p is arbitrary (d) $k=2$, $p=3$

Soln: (a) ∵ Matrices P and Y are of the orders $p \times k$ and $3 \times k$ respectively.

∴ matrix will be defined if $k=3$, consequently PY will be of the order $p \times k$. Matrices W and Y are of the orders $n \times 3$ and $3 \times k$ respectively.

Q.22. If $n=p$, then the order of the matrix $7X - 5Z$ is

(a) $p \times 2$ (b) $2 \times n$ (c) $n \times 3$ (d) $p \times n$.

Soln: Matrix X is of the order $2 \times n$

∴ matrix $7X$ is also of the same order.

Matrix Z is of the order $2 \times p$ i.e. $2 \times n$.

∴ matrix $5Z$ is also of the same order.

Now, both the matrices $7X$ and $5Z$ are of the order $2 \times n$.

Thus, matrix $7X - 5Z$ is well-defined and is of the order $2 \times n$.

So, the correct option is [B].